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Introduction to FACETS: A Many-Facet Rasch Model Computer Program

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Outline

- Principles of the Rasch Model
- Many-Facet Rasch Model
- Facets Software
- Several Examples
- The Importance of Connectivity
- Incomplete Data

The Principles of the Rasch Model

- Consider a typical mathematics exam with ten increasingly difficult items j, each scored as correct or incorrect

	items				
persons	j **= 1**	j **= 2**	j **= 3**	j **= 4**	j **= ...**
i **= 1**					
i **= 2**					
i **= 3**					
i **= 4**					
i **= ...**					

The Principles of the Rasch Model

- For each cell a probability of success can be calculated

	items				
persons	*j* = 1	*j* = 2	*j* = 3	*j* = 4	*j* = ...
i = 1	.99	.85	.81	.79	.65
i = 2	.91	.81	.71	.61	.51
i = 3	.85	.79	.69	...	...
i = 4	.79	...	...		
i = ...	.71	...			

- P(success) = function of *person ability* and *item difficulty*: F(θ_i, δ_j)

The Principles of the Rasch Model

$$P(x=1 \mid \theta_i, \delta_j) = \frac{e^{(\theta_i-\delta_j)}}{1+e^{(\theta_i-\delta_j)}}$$

$$\ln\left(\frac{P(success)}{1-p(success)}\right) = \theta_i - \delta_j$$

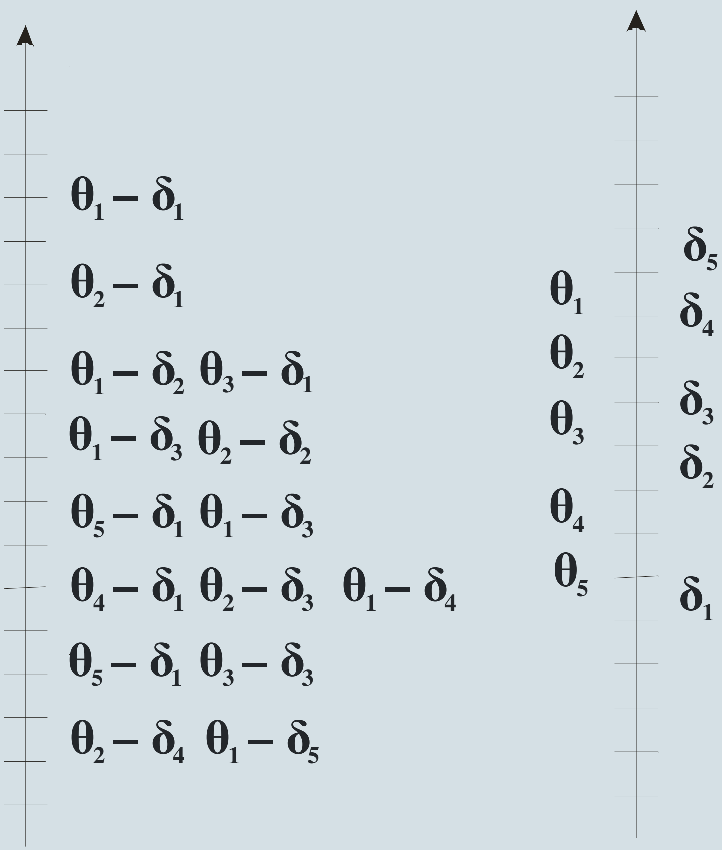
- Person ability (θ_i) and item difficulty (δ_j) are independent (i.e., additive)

The Principles of the Rasch Model

- Each datum can be expressed in terms of a combination of person ability and item difficulty

persons	items j = 1	j = 2	j = 3	j = 4	j = ...
i = 1	$\theta_1 - \delta_1$	$\theta_1 - \delta_2$	$\theta_1 - \delta_3$	$\theta_1 - \delta_4$	$\theta_1 - \delta_5$
i = 2	$\theta_2 - \delta_1$	$\theta_2 - \delta_2$	$\theta_2 - \delta_3$	$\theta_2 - \delta_4$	$\theta_2 - \delta_5$
i = 3	$\theta_3 - \delta_1$	$\theta_3 - \delta_2$	$\theta_3 - \delta_3$	..	..
i = 4	$\theta_4 - \delta_1$	..	..		
i = ...	$\theta_5 - \delta_1$	..			

The Principles of the Rasch Model



- item difficulty (δ_j) and person ability (θ_i) have the same units: Logits or log odds units
- If the data fit the Rasch model, we have:
  - A one-dimensional interval scale of the latent trait
  - Invariance between item difficulty and person ability

Many-Facet Rasch Model

- Traditional Rasch Model has two facets

- Many-Facet Model (multiple facets):

$$\ln\left(\frac{P(success)}{1-P(success)}\right)=\theta_i - R_r - T_t - \ldots\ldots - \delta_j$$

- Developed by John M. Linacre (1994)

FACETS: Some Figures

- Facets can handle up to:
  - 1.000.000 persons
  - 255 facets
  - 90% missing data
- Number of people currently using FACETS:
  - 400 single user licenses
  - 22 site licenses (4 in Europe; 1 in the Netherlands)
- Developed by John M. Linacre

FACETS: Different Types of Data

- Ordinal data
  - Dichotomous items
  - Multiple choice items
    - Andrich's rating scale ($D_{jk} = \delta_j + \tau_k$)
    - partial credit model ($D_{jk} = \delta_j + \tau_{jk}$)
  - Paired comparisons
- Continuous data
  - Counts, time, distance, etc.

EXAMPLES

Ex.1: 2-Facets with Dichotomous Data

- Field sobriety "Walk-the-Line" test on 15 suspects

- Seven items "yes/no" format
  - more or less than 9 steps
  - used arms for balance
  - too much swaying
  - failed to turn on one foot
  - did not walk a straight line
  - did not walk heal-to-toe
  - fell over during instructions

Ex.1: 2-Facets with Dichotomous Data

Data:

$\underbrace{1,1\text{-}7}_{\text{descriptors}},\underbrace{1,1,1,1,0,1,0}_{\text{data}}$; Row contains for person 1, the responses to items 1 to 7: yes, yes, yes, yes, no, yes, no

2,1-7,0,1,0,0,0,0,0 ; Row contains for person 2, the responses to items 1 to 7: no, yes, no, no, no, no, no

...

15,1-7,1,1,1,1,0,0,0

Ex.1: 2-Facets with Dichotomous Data

- Model:

$$P(x=1|\theta_i,\delta_j)=\frac{e^{(\theta_i-\delta_j)}}{1+e^{(\theta_i-\delta_j)}}$$

- The probability that a certain person *i* scores a "yes" on a certain behavior *j* (e.g., fail to walk heal-to-toe) is governed by:
  - the person's drunkenness (θ_i) MINUS
  - the easiness of performing a behavior (δ_j)

Ex.1: 2-Facets with Dichotomous Data

title=Example 1: 2-facet (traditional) Rash model......

facets=2 ; There are two facets (persons and items)

noncenter=1 ; Mean of facet 2 estimates are anchored at zero

positive=1 ; Facet 1 is positive (i.e., $+\theta_i - \delta_j$)

model=?,?,D ; Model with two facets (?s) and dichotomous data (D) **model statement should match dataset!**

Ex.1: 2-Facets with Dichotomous Data

```
Labels =

1,suspects     ; Facet 1 is called suspects (should match dataset!)
1=Julia C      ; Element 1 in facet 1 is Julia C
...
15=William D ; Element 15 in facet 1 is William D
*               ; End element list with * !

2,items          ; Facet 2 is called items
1=Took less or more than nine steps ; Label for item 1
...
7=Fell over during officer's instructions ;Label for item 2
*
```

Ex.1: 2-Facets with Dichotomous Data

```
Data =

1,1-7,1,1,1,1,0,1,0
2,1-7,0,1,0,0,0,0,0
3,1-7,1,1,1,0,0,0,0
4,1-7,1,0,0,0,0,0,0
...
...
...
14,1-7,1,1,1,1,1,0,0
15,1-7,1,1,1,1,0,0,0
→                         ; End control file with a → !
```

```
------------------------------------------------------------------------------------------------------
|Measr|+suspects                                           |-items                                   |
------------------------------------------------------------------------------------------------------
+   5 +                                                    + fell over during officer's instructions +
|     |                                                    |                                         |
|     |                                                    |                                         |
|     | Christina C  Marc H                                |                                         |
|     |                                                    |                                         |
+   4 +                                                    +                                         +
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    | Did not walk heal-to-toe                |
+   3 +                                                    +                                         +
|     |                                                    |                                         |
|     | Julia C      Julia H      Karl K                   |                                         |
|     |                                                    |                                         |
|     |                                                    |                                         |
+   2 +                                                    + Walked in other than a straight line    +
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    |                                         |
+   1 + Anthony P    Boris D      Chris I      William D   +                                         +
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    | Failed to turn on one feet              |
|     |                                                    |                                         |
*   0 *                                                    *                                         *
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    |                                         |
+  -1 + Ann H        Jennifer O   Lennard S                +                                         +
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    |                                         |
|     |                                                    |                                         |
+  -2 +                                                    +                                         +
|     |                                                    |                                         |
|     |                                                    | Too much swaying                        |
|     |                                                    |                                         |
|     | George M                                           |                                         |
+  -3 +                                                    +                                         +
|     |                                                    |                                         |
|     |                                                    | Took less or more than nine steps       |
|     |                                                    |                                         |
|     |                                                    |                                         |
+  -4 +                                                    +                                         +
|     |                                                    |                                         |
|     | Bill W       Jim J                                 |                                         |
|     |                                                    | Used arms for balance                   |
|     |                                                    |                                         |
------------------------------------------------------------------------------------------------------
|Measr|+suspects                                           |-items                                   |
------------------------------------------------------------------------------------------------------
```

Results

Table 6.0: Item-Person Map

Obsvd Score	Obsvd Count	Obsvd Average	Fair-M Avrage	Measure	Model S.E.	Infit MnSq	Infit ZStd	Outfit MnSq	Outfit ZStd	Estim. Discrm	Nu suspects
6	7	.9	.99	4.32	1.40	.36	-.9	.14	4.9	1.69	5 Christina C
6	7	.9	.99	4.32	1.40	1.99	1.2	.74	5.0	.13	12 Marc H
5	7	.7	.93	2.59	1.26	1.36	.7	.55	2.0	.76	1 Julia C
5	7	.7	.93	2.59	1.26	2.16	1.6	1.69	2.2	-.45	9 Karl K
5	7	.7	.93	2.59	1.26	.42	-.9	.19	1.9	1.67	14 Julia H
4	7	.6	.72	.97	1.32	.36	-.9	.15	.5	1.56	8 Chris I
4	7	.6	.72	.97	1.32	.36	-.9	.15	.5	1.56	10 Boris D
4	7	.6	.72	.97	1.32	1.68	1.0	.68	.8	.61	11 Anthony P
4	7	.6	.72	.97	1.32	.36	-.9	.15	.5	1.56	15 William D
3	7	.4	.29	-.91	1.44	.19	-.9	.10	.5	1.50	3 Jennifer O
3	7	.4	.29	-.91	1.44	.19	-.9	.10	.5	1.50	7 Ann H
3	7	.4	.29	-.91	1.44	.19	-.9	.10	.5	1.50	13 Lennard S
2	7	.3	.06	-2.72	1.26	2.47	1.9	3.49	2.6	-1.33	6 George M
1	7	.1	.01	-4.33	1.33	.47	-.9	.18	5.4	1.77	2 Jim J
1	7	.1	.01	-4.33	1.33	1.53	.9	.58	5.5	.40	4 Bill W
3.7	7.0	.5	.57	.41	1.34	.94	-.1	.60	2.3		Mean (Count: 15)
1.5	.0	.2	.36	2.66	.07	.80	1.1	.87	1.9		S.D. (Populn)
1.6	.0	.2	.37	2.75	.07	.83	1.1	.91	2.0		S.D. (Sample)

Model, Populn: RMSE 1.34 Adj (True) S.D. 2.29 Separation 1.71 Reliability .75
Model, Sample: RMSE 1.34 Adj (True) S.D. 2.40 Separation 1.79 Reliability .76
Model, Fixed (all same) chi-square: 59.4 d.f.: 14 significance (probability): .00
Model, Random (normal) chi-square: 17.3 d.f.: 13 significance (probability): .19

Table 7.1.1: Person Estimates and Fit Statistics

Obsvd Score	Obsvd Count	Obsvd Average	Fair-M Avrage	Measure	Model S.E.	Infit MnSq	Infit ZStd	Outfit MnSq	Outfit ZStd	Estim. Discrm	N items
1	15	.1	.01	4.98	1.15	.76	-.1	.19	3.4	1.24	7 fell over during officer's instructions
3	15	.2	.06	3.19	.83	1.12	.3	.54	1.3	.99	6 Did not walk heal-to-toe
5	15	.3	.17	1.97	.75	.81	-.4	.48	.5	1.30	5 Walked in other than a straight line
8	15	.5	.51	.35	.75	1.50	1.2	2.36	1.2	.17	4 Failed to turn on one feet
12	15	.8	.94	-2.39	.97	.29	-1.4	.12	.5	1.51	3 Too much swaying
13	15	.9	.98	-3.42	1.06	1.18	.4	.35	1.5	.99	1 Took less or more than nine steps
14	15	.9	.99	-4.69	1.22	.79	-.1	.16	3.1	1.26	2 Used arms for balance
8.0	15.0	.5	.52	.00	.96	.92	.0	.60	1.7		Mean (Count: 7)
4.8	.0	.3	.42	3.35	.18	.36	.8	.74	1.1		S.D. (Populn)
5.2	.0	.3	.45	3.62	.19	.39	.8	.79	1.2		S.D. (Sample)

Model, Populn: RMSE .98 Adj (True) S.D. 3.20 Separation 3.28 Reliability .92
Model, Sample: RMSE .98 Adj (True) S.D. 3.48 Separation 3.57 Reliability .93
Model, Fixed (all same) chi-square: 70.5 d.f.: 6 significance (probability): .00
Model, Random (normal) chi-square: 5.9 d.f.: 5 significance (probability): .32

Table 7.2.1: Item Estimates and Fit Statistics

Cat	Step	Exp.	Resd	StRes	Nu	suspects	N	items
1	1	.0	1.0	4	6	George M	4	Failed to turn on one feet
0	0	.9	-.9	-3	9	Karl K	4	Failed to turn on one feet
Cat	Step	Exp.	Resd	StRes	Nu	suspects	N	items

Table 4.1: Unexpected responses

Ex.2a: 3-Facets Including Different Tasks

- How sensitive is the "Walk-the-Line" test?
- Each participants performs 10 different tasks
  - drinking 1 beer
  - drinking 2 beers
  - ...
  - drinking 10 beers
- Each participant "walks the line" after each task

Ex.2a: 3-Facets Including Different Tasks

1,1,1-7,0,0,0,0,0,0,0	; row contains the scores of person 1 on task 1 for items 1 to 7
1,2,1-7,0,0,0,0,0,0,0	; row contains the scores of person 1 on task 2 for items 1 to 7
...	
1,10,1-7,1,1,0,1,1,1,0	; row contains the scores of person 1 on task 10 for items 1 to 7
2,1,1-7,0,0,0,0,0,0,0	; row contains the scores of person 2 on task 1 for items 1 to 7
....	
50,10,1-7,0,1,1,0,0,1,0	

Ex.2a: 3-Facets Including Different Tasks

- Model:

$$P(x=1|\theta_i, T_t, \delta_j) = \frac{e^{(\theta_i + T_t - \delta_j)}}{1 + e^{(\theta_i + T_t - \delta_j)}}$$

- The probability that a certain person *i* scores a "yes" on a certain behavior *j* (e.g., fail to walk heal-to-toe) after a certain amount *t* of beer is governed by:
  - the person's sensitivity to alcohol (θ_i) PLUS
  - the amount of alcohol intake in the task (T_t) MINUS
  - the easiness of the behavior (δ_j)

Ex.2a: 3-Facets Including Different Tasks

title=Example 2a: 3-facet model - 10 tasks...

facets=3 ; This time three facets

noncenter=1 ; Means of facets 2 and 3 are at zero

positive=1,2 ; Facet 1 and 2 are positive (i.e., $+\theta_i + T_t - \delta_j$)

models=?,?,?,D ;three facets (?s) to be estimated

Ex.2a: 3-Facets Including Different Tasks

```
Labels =

1,persons
1-50=                    ; no labels, just 50 participants
*

2,tasks                  ; Facet 2 contains the tasks
1=1 consumption
2=2 consumptions
...
10=10 consumptions
*
```

Ex.2a: 3-Facets Including Different Tasks

```
3,items        ;Facet 3 now contains the items
1=Took less or more than nine steps
...
7=fell over during officer's instructions
*

Data=

1,1,1-7,0,0,0,0,0,0,0
1,2,1-7,0,0,0,0,0,0,0
...
50,10,1-7,0,1,1,0,0,1,0
→
```

Results

Table 6.0:

Facet Map

```
-------------------------------------------------------------------------------------------------
|Measr|+persons |+tasks                            |-items                                   |
-------------------------------------------------------------------------------------------------
|     |         |                                  | fell over during officer's instructions |
|     |         |                                  |                                         |
|     |         |                                  |                                         |
+   2 +         +                                  +                                         +
|     |         |                                  |                                         |
|     |         |                                  |                                         |
|     |         | 10 consumptions                  | Did not walk heal-to-toe                |
|     |         |                                  | Walked in other than a straight line    |
+   1 +         + 9 consumptions                   +                                         +
|     |         | 8 consumptions                   |                                         |
|     | ***     |                                  |                                         |
|     | *       |                                  |                                         |
|     | ***     |                                  |                                         |
*   0 * *       * 7 consumptions                   *                                         *
|     | ****    | 6 consumptions                   |                                         |
|     | *       | 5 consumptions                   | Failed to turn on one feet              |
|     | ****    |                                  |                                         |
|     | *       |                                  |                                         |
+  -1 +         +                                  + Too much swaying                        +
|     | ******  | 4 consumptions                   |                                         |
|     | *       |                                  |                                         |
|     | ***     | 3 consumptions                   | Used arms for balance                   |
|     |         |                                  |                                         |
+  -2 + ****    +                                  + Took less or more than nine steps       +
|     | ******* |                                  |                                         |
|     | **      |                                  |                                         |
|     |         |                                  |                                         |
|     | **      |                                  |                                         |
+  -3 + **      +                                  +                                         +
|     | *       |                                  |                                         |
|     |         |                                  |                                         |
|     |         |                                  |                                         |
|     |         |                                  |                                         |
+  -4 + *       +                                  +                                         +
|     |         |                                  |                                         |
|     |         |                                  |                                         |
|     | **      |                                  |                                         |
|     |         |                                  |                                         |
+  -5 +         +                                  +                                         +
|     |         |                                  |                                         |
|     | *       |                                  |                                         |
|     |         |                                  |                                         |
|     |         |                                  |                                         |
+  -6 +         + 1 consumption    2 consumptions  +                                         +
-------------------------------------------------------------------------------------------------
|Measr| * = 1   |+tasks                            |-items                                   |
-------------------------------------------------------------------------------------------------
```

Obsvd Score	Obsvd Count	Obsvd Average	Fair-M Avrage	Measure	Model S.E.	Infit MnSq	Infit ZStd	Outfit MnSq	Outfit ZStd	Estim. Discrm	Nu	tasks
171	350	.5	.46	1.39	.14	.91	-1.3	1.02	.1	1.09	10	10 consumptions
156	350	.4	.39	1.08	.14	1.01	.1	.89	-.4	1.01	9	9 consumptions
146	350	.4	.34	.88	.14	1.06	.9	1.33	1.3	.88	8	8 consumptions
107	350	.3	.18	.06	.15	.88	-1.6	.62	-1.1	1.19	7	7 consumptions
94	350	.3	.14	-.23	.15	1.03	.3	1.38	.9	.94	6	6 consumptions
84	350	.2	.12	-.47	.16	.99	.0	1.44	.9	.97	5	5 consumptions
60	350	.2	.07	-1.10	.17	1.04	.4	.75	-.2	.97	4	4 consumptions
44	350	.1	.04	-1.61	.19	.99	.0	.99	.2	1.01	3	3 consumptions
0	350	.0	.00	(-7.17	1.82)	Minimum					1	1 consumption
0	350	.0	.00	(-7.17	1.82)	Minimum					2	2 consumptions
86.2	350.0	.2	.17	-1.43	.49	.99	-.1	1.05	.2		Mean	(Count: 10)
57.8	.0	.2	.16	3.00	.67	.06	.8	.29	.8		S.D.	(Populn)
61.0	.0	.2	.17	3.16	.70	.06	.9	.31	.8		S.D.	(Sample)

With extremes, Model, Populn: RMSE .83 Adj (True) S.D. 2.89 Separation 3.49 Reliability .92
With extremes, Model, Sample: RMSE .83 Adj (True) S.D. 3.06 Separation 3.70 Reliability .93
Without extremes, Model, Populn: RMSE .16 Adj (True) S.D. .99 Separation 6.32 Reliability .98
Without extremes, Model, Sample: RMSE .16 Adj (True) S.D. 1.06 Separation 6.76 Reliability .98
With extremes, Model, Fixed (all same) chi-square: 341.4 d.f.: 9 significance (probability):.00
With extremes, Model, Random (normal) chi-square: 7.6 d.f.: 8 significance (probability): .48

Table 7.2.1: Task Estimates and Fit Statistics

Ex.2b: 3-Facets Including Different Raters

- Audio-visual recordings of 10 job interviews
- 3 Judges rate each person's communicational skill by means of 5 items on a 5 point scale (ranging from 0 = weak to 4 = strong)
- 5 items:
  – general appearance
  – non-verbal skills
  – verbal skill
  – Pronunciation
  – confidence

Ex.2b: 3-Facets Including Different Raters

Data set

1,1,1-5,4,3,4,4,4

1,2,1-5,4,4,4,0,0

1,3,1-5,3,4,3,4,4

2,1,1-5,3,3,3,3,0

...

...

10,3,1-5,3,3,0,0,0

Ex.2b: 3-Facets Including Different Raters

- Model:

$$P(x=1|\theta_i, R_r, D_{jk}) = \frac{e^{(\theta_i - R_r - D_{jk})}}{1+e^{(\theta_i - R_r - D_{jk})}}$$

- The probability that a certain person *i* receives a certain score *k* (e.g., 4) on a certain item *j* (e.g., nonverbal skills) by a certain rater *r* is governed by:
  - the person's communication skill (θ_i) MINUS
  - the harshness of the rater (R_r) MINUS
  - the difficulty of receiving a *k* on item *j* (D_{jk})

Ex.2b: 3-Facets Including Different Raters

- We will be using Andrich's rating scale to model the step difficulties:
  - A mean item difficulty ($\boldsymbol{\delta}_j$) is estimated for each item
  - All item share the same step difficulties ($\boldsymbol{\tau}_k$)

- So that: $D_{jk} = \boldsymbol{\delta}_j + \boldsymbol{\tau}_k$

Ex.2b: 3-Facets Including Different Raters

```
title=Example 2b: Job Application Interview - 3 Judges ....
facets=3                          ; again three facets
noncenter=1
positive=1                        ; only facet 1 is positive (i.e., + θi - Rr - δj)
models=?,?,?,ComSkill             ; ComSkill is the name of the scale
Rating scale = ComSkill,R5        ; each item has 5 levels (R5)
0=very weak                       ; 0 = labeled very weak
1=weak                            ; 1 = labeled weak
2=average                         ; etc.
3=good                            ; etc.
4=very good                       ; etc.
*                                 ; again end label list with a *
```

Ex.2b: 3-Facets Including Different Raters

```
Labels =
1,persons
1=Peter
...
*
2,Rater
1=JudgeA
...
*
3,Items
1=general appearance
...
*
```

Results

Table 6.0: Facet Map

```
-------------------------------------------------------
|Measr|+persons    |-Rater  |-Items              |COMSK|
-------------------------------------------------------
|     |           |        |                    |     |
|     | Christina |        |                    |     |
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           |        |                    |     |
+   2 +           +        +                    +     +
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           |        |                    | --- |
|     |           |        |                    |     |
|     | Peter     |        |                    |     |
+   1 +           +        +                    +     +
|     |           |        |                    |     |
|     | Marcel    |        | confidence         |     |
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           |        | Pronunciation      |     |
|     | Jennifer  | JudgeB |                    |  3  |
|     |           |        |                    |     |
|     |           |        |                    |     |
|     |           | JudgeC | verbal skill       |     |
*   0 * George    *        *                    * --- *
|     |           |        |                    |     |
|     | Julia     |        |                    |  2  |
|     |           |        |                    |     |
|     |           |        |                    | --- |
|     | William   | JudgeA |                    |     |
|     |           |        | non-verbal skills  |     |
|     | Bill      |        |                    |  1  |
|     |           |        | general appearance |     |
|     |           |        |                    |     |
+  -1 + Ann       +        +                    +     +
|     | Jim       |        |                    | --- |
|     |           |        |                    |     |
-------------------------------------------------------
|Measr|+persons    |-Rater  |-Items              |COMSK|
-------------------------------------------------------
```

Obsvd Score	Obsvd Count	Obsvd Average	Fair-M Avrage	Measure	Model S.E.	Infit MnSq	Infit ZStd	Outfit MnSq	Outfit ZStd	Estim. Discrm	N Items
39	30	1.3	1.09	.80	.19	1.19	.6	.77	-.2	1.01	5 confidence
47	30	1.6	1.64	.53	.18	.90	-.2	.68	-.6	1.29	4 Pronunciation
62	30	2.1	2.55	.07	.17	.89	-.3	1.03	.1	.54	3 verbal skill
83	30	2.8	3.23	-.57	.18	.68	-1.1	.71	-.7	1.02	2 non-verbal skills
90	30	3.0	3.37	-.82	.19	.80	-.6	1.40	1.0	1.04	1 general appearance
64.2	30.0	2.1	2.37	.00	.18	.89	-.3	.92	-.1		Mean (Count: 5)
19.8	.0	.7	.89	.62	.01	.17	.6	.27	.7		S.D. (Populn)
22.1	.0	.7	.99	.69	.01	.19	.7	.30	.7		S.D. (Sample)

Model, Populn: RMSE .18 Adj (True) S.D. .59 Separation 3.24 Reliability .91
Model, Sample: RMSE .18 Adj (True) S.D. .67 Separation 3.65 Reliability .93
Model, Fixed (all same) chi-square: 54.6 d.f.: 4 significance (probability): .00
Model, Random (normal) chi-square: 3.7 d.f.: 3 significance (probability): .29

Table 7.3.1: Item Estimates and Fit Statistics

Mean item difficulty $\boldsymbol{\delta}_j$

Obsvd Score	Obsvd Count	Obsvd Average	Fair-M Avrage	Measure	Model S.E.	Infit MnSq	Infit ZStd	Outfit MnSq	Outfit ZStd	Estim. Discrm	N Items
39	30	1.3	1.09	.80	.19	1.19	.6	.77	-.2	1.01	5 confidence
47	30	1.6	1.64	.53	.18	.90	-.2	.68	-.6	1.29	4 Pronunciation
62	30	2.1	2.55	.07	.17	.89	-.3	1.03	.1	.54	3 verbal skill
83	30	2.8	3.23	-.57	.18	.68	-1.1	.71	-.7	1.02	2 non-verbal skills
90	30	3.0	3.37	-.82	.19	.80	-.6	1.40	1.0	1.04	1 general appearance
64.2	30.0	2.1	2.37	.00	.18	.89	-.3	.92	-.1		Mean (Count: 5)
19.8	.0	.7	.89	.62	.01	.17	.6	.27	.7		S.D. (Populn)
22.1	.0	.7	.99	.69	.01	.19	.7	.30	.7		S.D. (Sample)

Model, Populn: RMSE .18 Adj (True) S.D. .59 Separation 3.24 Reliability .91
Model, Sample: RMSE .18 Adj (True) S.D. .67 Separation 3.65 Reliability .93
Model, Fixed (all same) chi-square: 54.6 d.f.: 4 significance (probability): .00
Model, Random (normal) chi-square: 3.7 d.f.: 3 significance (probability): .29

Table 7.3.1: Item Estimates and Fit Statistics

DATA				QUALITY CONTROL			STEP		EXPECTATION	
Category Score	Counts Used	%	Cum. %	Avge Meas	Exp. Meas	OUTFIT MnSq	CALIBRATIONS Measure	S.E.	Measure at Category	-0.5
0	47	31%	31%	-1.14	-1.04	.6			(-1.44)	
1	11	7%	39%	-.50	-.61	1.1	.62	.25	-.70	-1.08
2	5	3%	42%	.05	-.15	.4	.40	.25	-.21	-.45
3	48	32%	74%	.73	.43	1.1	-2.14	.25	.43	.05
4	39	26%	100%	1.15	1.46	1.2	1.11	.24	(2.23)	1.29
									(Mean)	

Table 8.1: Item step statistics

step difficulties τ_k

DATA Category Score	Counts Used	%	Cum. %	QUALITY CONTROL Avge Meas	Exp. Meas	OUTFIT MnSq	STEP CALIBRATIONS Measure	S.E.	EXPECTATION Measure at Category	-0.5
0	47	31%	31%	-1.14	-1.04	.6			(-1.44)	
1	11	7%	39%	-.50	-.61	1.1	.62	.25	-.70	-1.08
2	5	3%	42%	.05	-.15	.4	.40	.25	-.21	-.45
3	48	32%	74%	.73	.43	1.1	-2.14	.25	.43	.05
4	39	26%	100%	1.15	1.46	1.2	1.11	.24	(2.23)	1.29
									(Mean)	

Table 8.1: Item step statistics

Ex.2b: 3-Facets Including Different Raters

- IMPORTANT:
- There is *formally* no difference between a facet containing persons, items, tasks or raters
- Getting a high score on an easy task is similar to receiving a high score from a lenient judge

Ex.2c: 3-Facets Including Rater Bias

- Same data and model as in Example 2b

- However this time we test for possible bias in a rater's evaluation of a specific job applicant

```
model=?B,?B,?,ComSkill   ; FACETS now tests for biases between
                           facet 1 (persons) and facet 2 (raters)
```

Ex.2c: 3-Facets Including Rater Bias

- Pairwise (applicant by judge) comparison of estimated and observed scores

- Each specific bias (or interaction) between persons and raters is tested by exploring the residuals (i.e., the unexplained data)

Target Nu	persons	Target Measr	S.E.	Obs-Exp Average	Context N	Rater	Target Measr	S.E.	Obs-Exp Average	Context N	Rater	Target Contrast	Joint S.E.	t	d.f.	Prob.
9	George	1.26	.82	.83	1	JudgeA	-1.44	.53	-2.37	3	JudgeC	2.69	.98	2.75	8	.0249
9	George	.56	.39	.79	2	JudgeB	-1.44	.53	-1.21	3	JudgeC	2.00	.66	3.03	8	.0163
1	Peter	2.17	1.12	.26	1	JudgeA	.56	.39	-1.14	2	JudgeB	1.61	1.18	1.36	8	.2106
8	Christina	4.29	1.78	.26	2	JudgeB	2.73	1.12	.06	3	JudgeC	1.55	2.10	.74	8	.4817
3	Jim	-1.23	.40	-.12	1	JudgeA	-2.29	1.58	-1.32	2	JudgeB	1.06	1.63	.65	8	.5344
5	Jennifer	.74	.63	.25	1	JudgeA	-.19	.40	-1.75	2	JudgeB	.93	.74	1.25	8	.2467
6	Marcel	1.29	.51	.48	2	JudgeB	.40	.40	-.12	3	JudgeC	.89	.65	1.38	8	.2062
9	George	1.26	.82	.83	1	JudgeA	.56	.39	-.37	2	JudgeB	.70	.91	.77	8	.4643

Table 14.1.1.2: Bias / interaction pairwise report

The Importance of Connectivity

- Up to now all examples had complete data sets:
  - All persons were scored on all tasks or were rated by all judges
- Complete data sets are often infeasible
- The FACETS software can handle incomplete data
- But only if there is sufficient connectivity between the different data points

When Is There No Connectivity?

	Judges		
	1	2	3
1	X		
2	X		
3		X	
4		X	
5			X
6			X

- Each person is rated by one judge
- There are three subsets that cannot be compared with each other
- No connectivity!
- FACETS will produce a different scale for each of the three subsets

Two Examples of Connectivity

	Judges 1	Judges 2	Judges 3
1	X		X
2	X		X
3		X	X
4		X	X
5			X
6			X

	Judges 1	Judges 2	Judges 3
1	X	X	
2		X	X
3	X		X
4	X		X
5	X	X	
6		X	X

rotated judgment plan

Ex.3: Incomplete Data

- As in experiment 2b and 2c: Audio-visual recordings of 10 persons' job interviews
- Each person is rated by two judges according to a rotated judgment plan
- On communicational skills by means of 5 items on a 5 point scale

Ex.3: Incomplete Data

Data

1,1,1-5,4,3,4,4,4
1,2,1-5,4,4,4,0,0
2,2,1-5,3,3,1,0,0
2,3,1-5,3,3,4,0,0
3,1,1-5,3,3,0,0,0
3,3,1-5,3,3,0,0,0
...

Obsvd Score	Obsvd Count	Obsvd Average	Fair-M Avrage	Measure	Model S.E.	Infit MnSq	Infit ZStd	Outfit MnSq	Outfit ZStd	Estim. Discrm	Nu persons
39	10	3.9	3.94	3.98	1.07	1.11	.4	1.24	.5	.89	8 Christina
31	10	3.1	3.34	.95	.36	1.30	.7	1.10	.3	1.05	1 Peter
30	10	3.0	3.10	.56	.34	1.35	.7	1.31	.6	.17	6 Marcel
22	10	2.2	2.99	.43	.28	.91	.0	.71	-.4	.79	5 Jennifer
17	10	1.7	2.49	.03	.29	.56	-.9	.40	-1.0	1.37	2 Julia
20	10	2.0	1.67	-.38	.29	1.67	1.4	1.48	.8	.33	9 George
16	10	1.6	1.29	-.57	.30	.35	-1.9	.63	-.2	.71	4 William
13	10	1.3	.81	-.84	.31	.72	-.4	.45	-.3	1.47	7 Bill
12	10	1.2	.52	-1.08	.31	.76	-.3	.51	-.3	1.52	3 Jim
8	10	.8	.30	-1.39	.36	.72	-.2	.41	-.1	1.13	10 Ann
20.8	10.0	2.1	2.05	.17	.39	.94	-.1	.82	.0		Mean (Count: 10)
9.3	.0	.9	1.23	1.46	.23	.39	.9	.39	.6		S.D. (Populn)
9.8	.0	1.0	1.29	1.54	.24	.41	1.0	.41	.6		S.D. (Sample)

Model, Populn: RMSE .45 Adj (True) S.D. 1.38 Separation 3.06 Reliability .90
Model, Sample: RMSE .45 Adj (True) S.D. 1.47 Separation 3.25 Reliability .91
Model, Fixed (all same) chi-square: 61.6 d.f.: 9 significance (probability): .00
Model, Random (normal) chi-square: 6.9 d.f.: 8 significance (probability): .55

Table 7.1.1: Person Estimates and Fit Statistics

Obsvd Score	Obsvd Count	Obsvd Average	Fair-M Avrage	Measure	Model S.E.	Infit MnSq	Infit ZStd	Outfit MnSq	Outfit ZStd	Estim. Discrm	Nu	persons
39	10	3.9	3.94	3.98	1.07	1.11	.4	1.24	.5	.89	8	Christina
31	10	3.1	3.34	.95	.36	1.30	.7	1.10	.3	1.05	1	Peter
30	10	3.0	3.10	.56	.34	1.35	.7	1.31	.6	.17	6	Marcel
22	10	2.2	2.99	.43	.28	.91	.0	.71	-.4	.79	5	Jennifer
17	10	1.7	2.49	.03	.29	.56	-.9	.40	-1.0	1.37	2	Julia
20	10	2.0	1.67	-.38	.29	1.67	1.4	1.48	.8	.33	9	George
16	10	1.6	1.29	-.57	.30	.35	-1.9	.63	-.2	.71	4	William
13	10	1.3	.81	-.84	.31	.72	-.4	.45	-.3	1.47	7	Bill
12	10	1.2	.52	-1.08	.31	.76	-.3	.51	-.3	1.52	3	Jim
8	10	.8	.30	-1.39	.36	.72	-.2	.41	-.1	1.13	10	Ann
20.8	10.0	2.1	2.05	.17	.39	.94	-.1	.82	.0		Mean (Count: 10)	
9.3	.0	.9	1.23	1.46	.23	.39	.9	.39	.6		S.D. (Populn)	
9.8	.0	1.0	1.29	1.54	.24	.41	1.0	.41	.6		S.D. (Sample)	

Model, Populn: RMSE .45 Adj (True) S.D. 1.38 Separation 3.06 Reliability .90
Model, Sample: RMSE .45 Adj (True) S.D. 1.47 Separation 3.25 Reliability .91
Model, Fixed (all same) chi-square: 61.6 d.f.: 9 significance (probability): .00
Model, Random (normal) chi-square: 6.9 d.f.: 8 significance (probability): .55

Table 7.1.1: Person Estimates and Fit Statistics

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